

Estimation of a High Dimensional Vector of Means. A Non-Bayesian Empirical Bayes Approach.

Eitan Greenshtein

August 29, 2006

Abstract

We study the problem of estimating a high dimensional vector (μ_1, \dots, μ_n) based on independent $Y_i \sim N(\mu_i, 1)$, $i = 1, \dots, n$, under a square loss.

The idea is to try to imitate an ‘oracle’ that knows the ordered vector $(\mu_{(1)}, \dots, \mu_{(n)})$.

We derive a novel method. The ratio of its risk with the risk obtained by the above oracle, approaches 1 as n approaches infinity. This is under conditions where the oracle does not have extreme advantage. Such an extreme advantage is, e.g., when $\mu_i = 0$, $i = 1, \dots, n$ and whence knowing the ordered vector is equivalent to knowing the vector itself.

When the vector (μ_1, \dots, μ_n) is not extremely sparse, the performance of our method in simulations is very good in comparison to existing methods.