

# Considerations of Modeling in Keyword Bidding (Google:AdWords)

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# Outline

- I. Problem Description
- II. Game theoretical aspect of the bidding problem that we are considering
- III. Statistical Implementation
- IV. Conclusion

# I. Introduction

- We consider the modeling issue in AdWords
- An equilibrium strategy is derived, which we argue will be the foundation of statistical modeling
- What is AdWords: A tool at Google that allow users to bid for advertising positions at [google.com](http://google.com)



# dehumidifier

About 23,400,000 results (0.21 seconds)

## Organic search

Ad related to dehumidifier

Why this ad?

### [Top Rated Dehumidifiers - Ratings, Reviews & Many On Sale.](#)

[www.allergybuyersclub.com/](http://www.allergybuyersclub.com/) - 304 seller reviews  
Great Brands. Free Shipping. Save.

[Dehumidifiers](#) [Steam Cleaners](#)  
[Air Purifiers](#) [Hepa Vacuums](#)

### [Amazon.com: Dehumidifiers](#)

[www.amazon.com/b?ie=UTF8&node=267557011](http://www.amazon.com/b?ie=UTF8&node=267557011)

Results 1 - 24 of 584 – Online shopping for **Dehumidifiers** from a great selection of Appliances; & more at everyday low prices.

### [Dehumidifier - Wikipedia, the free encyclopedia](#)

[en.wikipedia.org/wiki/Dehumidifier](http://en.wikipedia.org/wiki/Dehumidifier)

A **dehumidifier** is typically a household appliance that reduces the level of humidity in the air, usually for health reasons. Humid air can cause mold and mildew ...

### [Top Dehumidifier Reviews | Best Dehumidifier – Cons...](#)

[www.consumerreports.org](http://www.consumerreports.org) › Home › Appliances

Looking for the Best **dehumidifier**? Consumer Reports has honest Ratings and Reviews on **dehumidifiers** from the unbiased experts you can trust.

### [Dehumidifiers & Energy Star Dehumidifier | Best Buy](#)

[www.bestbuy.com](http://www.bestbuy.com) › Appliances › Air Purifiers & Dehumidifiers

Shop online for **Dehumidifiers** at BestBuy.com for an Energy Star **Dehumidifier** and have it shipped or pick up in store!

### [Dehumidifiers - Walmart](#)

[www.walmart.com/cp/Dehumidifiers/112918](http://www.walmart.com/cp/Dehumidifiers/112918)

## AdWords

Ads - Why these ads?

### [Dehumidifier at Sears®](#)

[www.sears.com/Dehumidifiers](http://www.sears.com/Dehumidifiers)

3,675 reviews for sears  
Save on **Dehumidifiers** at Sea  
Shop Our Great Selection Toda

### [Dehumidifiers at Amazon](#)

[www.amazon.com/](http://www.amazon.com/)

8,178 seller reviews  
Buy **dehumidifiers** at Amazon!  
Qualified orders over \$25 ship fi

### [Top-Rated Dehumidifiers](#)

[www.sylvane.com/Dehumidifie](http://www.sylvane.com/Dehumidifie)

145 reviews for sylvane  
Compare & Review All Major Br  
Fast Free Shipping. 30-Day Ret

### [Which Dehumidifier to Buy](#)

[www.consumersearch.com/deh](http://www.consumersearch.com/deh)

We do the research so you don'  
have to. **Dehumidifier** Reviews

### [Dehumidifiers On Sale](#)

[www.compactappliance.com/De](http://www.compactappliance.com/De)

1,637 seller reviews  
20% Off ALL **Dehumidifiers** Sa  
Get Free Shipping & No Sales T

### [Basement Dehumidifier](#)

[www.walmart.com/Appliances](http://www.walmart.com/Appliances)

985 reviews for walmar

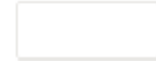
hotel raleigh nc

# Hotel raleigh NC



Sign in

About 6,230,000 results (0.35 seconds)



Ads related to hotel raleigh nc

[Why these ads?](#)

[Washington Duke Inn - Located at Duke University](#)

[www.washingtondukeinn.com/](http://www.washingtondukeinn.com/)

3 Night Special from \$119. Book Now

[Clarion Hotel® Raleigh NC - Official site |](#)

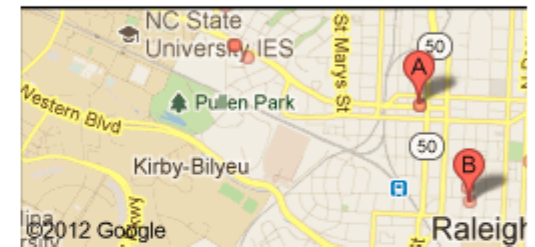
[ClarionHotel.com](http://ClarionHotel.com)

[www.clarionhotel.com/](http://www.clarionhotel.com/)

**Hotel** near Conventio Center. Best Internet Rates.

Summer Promo - Earn A \$50 Gift Card - Want Hotel Deals? Find Them Here

## Map for hotel raleigh nc



Ads - [Why these ads?](#)

[HYATT house® Raleigh](#)

[house.hyatt.com/](http://house.hyatt.com/)

All Suites **Hotels** with a Full Kitchen. Free Breakfast & Intern

122,198 people +1'd or follow [Hyatt](#)

[Hotels in Raleigh, NC](#)

[www.expedia.com/Raleigh\\_No](http://www.expedia.com/Raleigh_No)

436 reviews for expedia

Summer Sale: **Hotels** up to 30% Save on **Raleigh, NC Hotels**.

295,202 people +1'd or follow [Expedia](#)

[Book hotels in Raleigh](#)

[www.google.com/hotelfinder](http://www.google.com/hotelfinder)

2-star hotels from \$42     4-star hotels from \$179

3-star hotels from \$61     5-star hotels

[Book hotels](#)

Sponsored ⓘ

[Downtown Raleigh Hotels, Raleigh...](#)

320 Hillsborough Street  
Raleigh

# A Simplified Version of AdWords Bidding

- Sorted (descending) bids  $b_j, j = 1, 2, \dots, N$ , of  $N$  potential advertisers:  $b_1 > b_2 > \dots > b_N$
- Positions and Cost per Click by AdWords
- GSP: generalized second price

| Position | Bid price | Actual price (i.e., CPC) |
|----------|-----------|--------------------------|
| 1        | $b_1$     | $b_2 + 1$ cent           |
| 2        | $b_2$     | $b_3 + 1$ cent           |
| 3        | $b_3$     | $b_4 + 1$ cent           |
| 4        | $b_4$     | $b_5 + 1$ cent           |
| 5        | $b_5$     | Assume out of space      |

# Actual Scheme: Incorporate a Quality Score

- The quality score (aka, AdRank) that depends on relevance, past click through rate, landing page, etc.
- Purpose: integrating web page quality, user experience, user satisfaction
- Prevent: bad, irrelevant ads goes to top positions by paying more (customer satisfaction)

# An Example

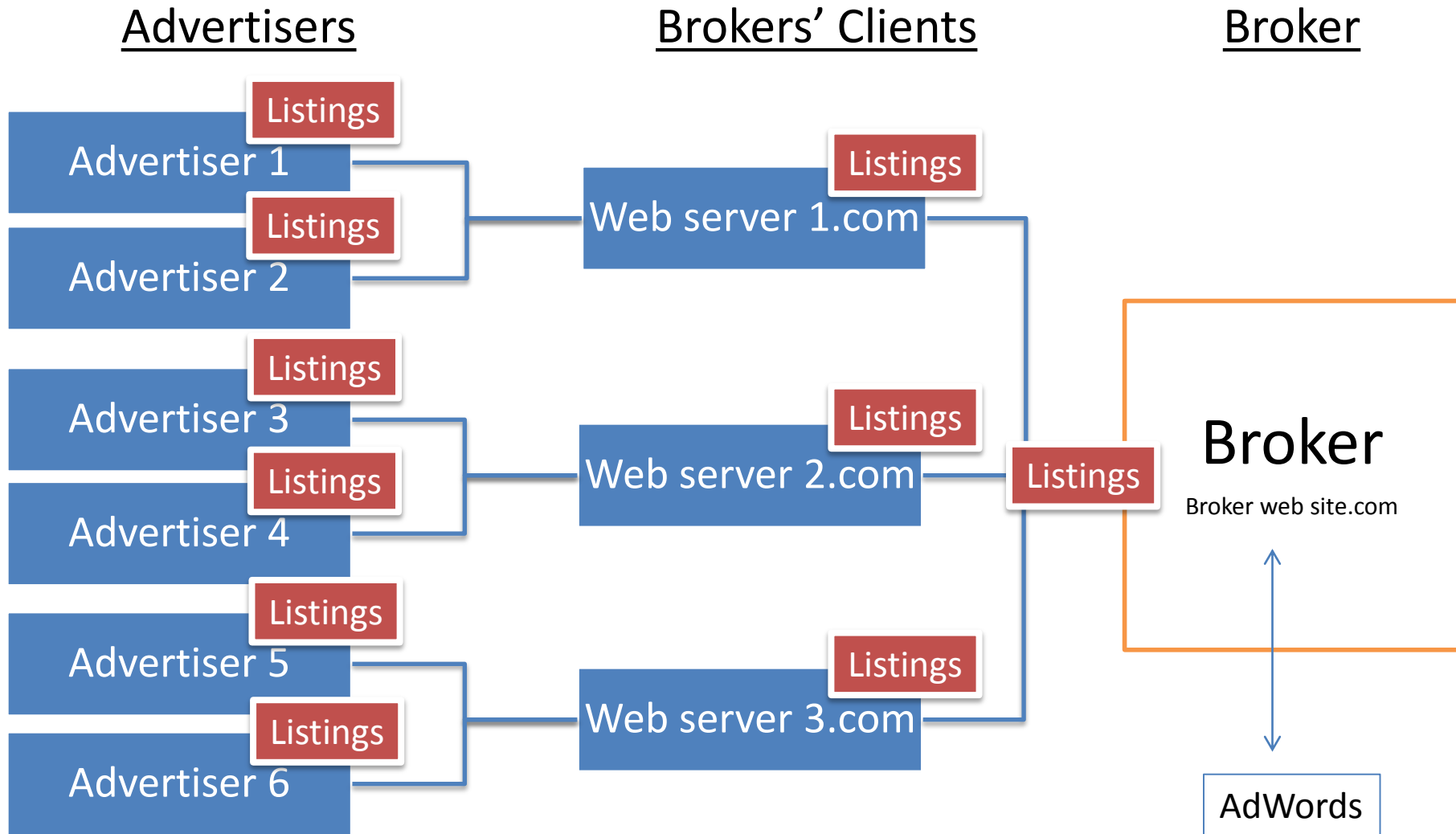
- Bidding with quality scores (rank according to  $b_j q_j$ )

| Position | Bid price | Quality score | Multiply  | Actual price (i.e., CPC) |
|----------|-----------|---------------|-----------|--------------------------|
| 1        | $b_1$     | $q_1$         | $b_1 q_1$ | $b_2 q_2 / q_1 + 1$ cent |
| 2        | $b_2$     | $q_2$         | $b_2 q_2$ | $b_3 q_3 / q_2 + 1$ cent |
| 3        | $b_3$     | $q_3$         | $b_3 q_3$ | $b_4 q_4 / q_3 + 1$ cent |
| 4        | $b_4$     | $q_4$         | $b_4 q_4$ | $b_5 q_5 / q_4 + 1$ cent |
| 5        | $b_5$     | $q_5$         | $b_5 q_5$ | Assume out of space      |

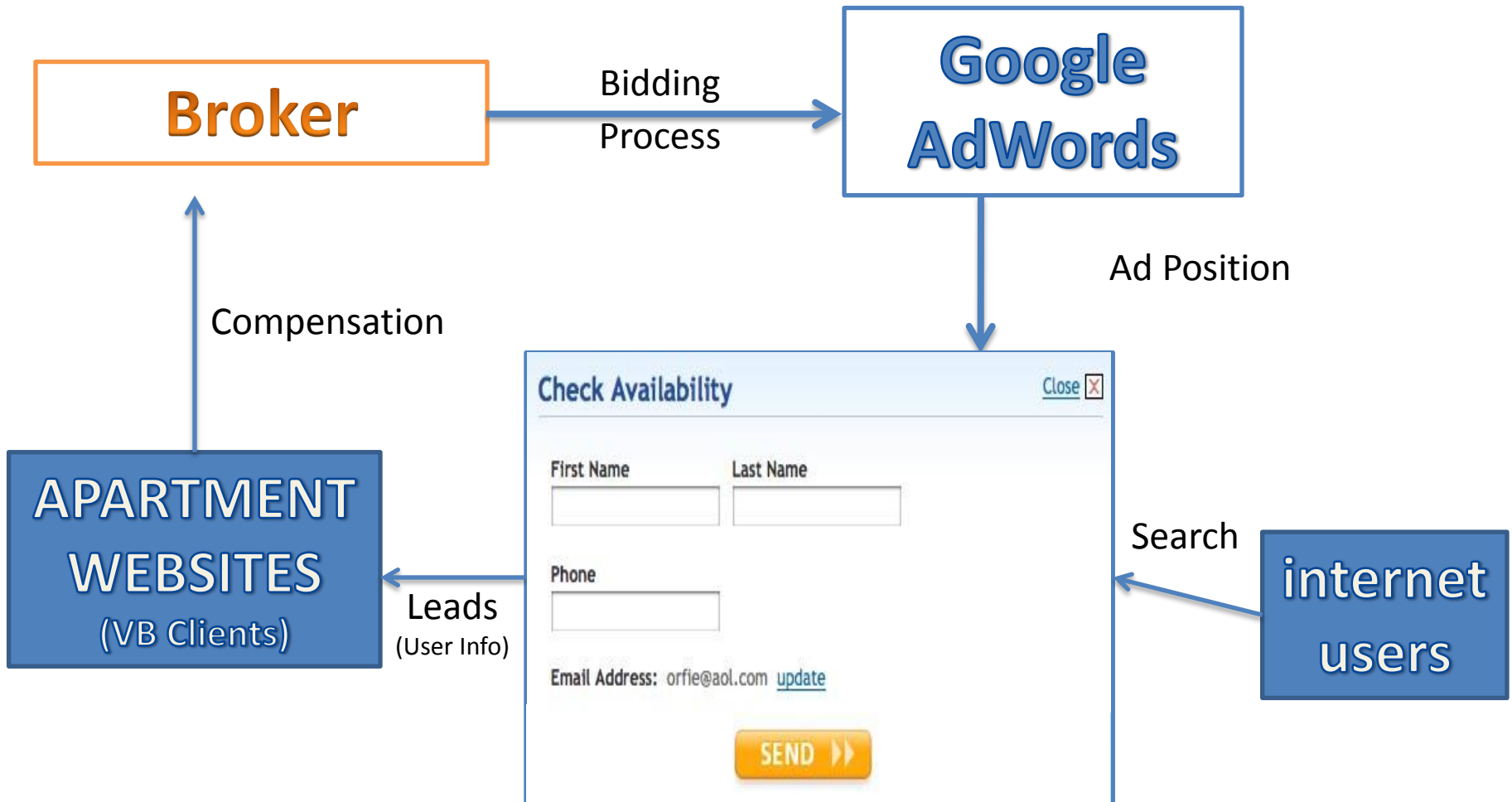
- Assume:  $b_1 q_1 > b_2 q_2 > \dots > b_N q_N$
- Note: the order of advertisers may change from the previous example



# Problem Description



# Broker & Google AdWords



# Review of a Few Terms

Impression

A link showed up

Click

Link is clicked

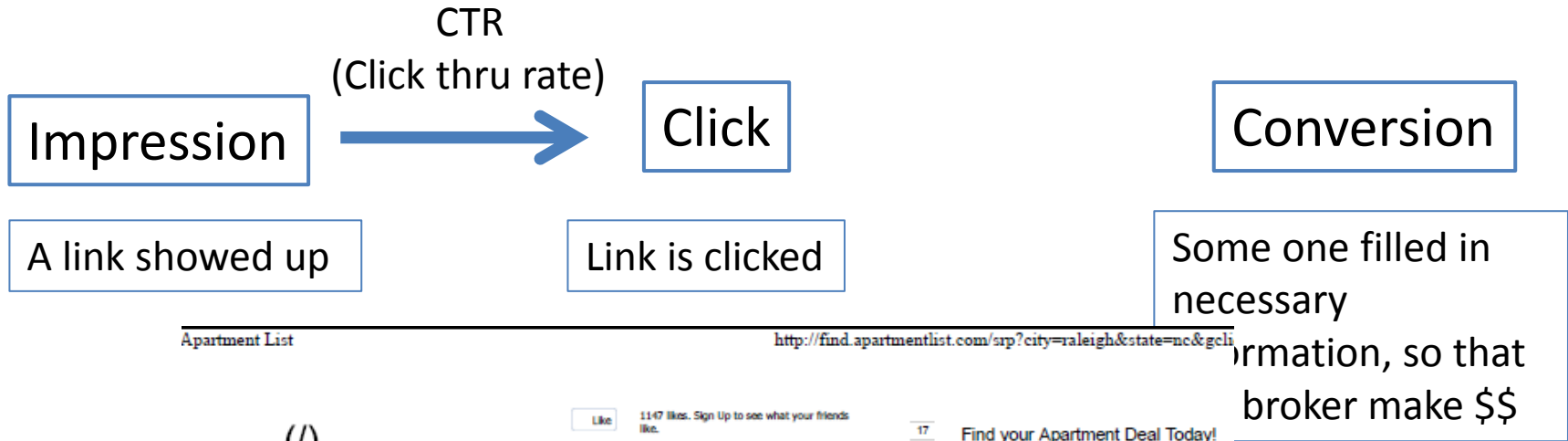
Conversion

Some one filled in necessary information, so that the broker make \$\$

The screenshot shows a search engine interface with the following elements:

- Search bar: "three bedroom apartments in raleigh nc"
- User profile: "Xiaom..."
- Results: "About 804,000 results (0.23 seconds)"
- Left column of ads:
  - Ad 1: "Find **Apartments** for Rent - Free Local **Apartment Search**. [www.apartmentguide.com/](http://www.apartmentguide.com/) View floorplans & virtual tours!"
  - Ad 2: "1.2 & 3 **Bedroom Apts** - For rent in the new Sterling [www.sterlingtowncenterapts.com/](http://www.sterlingtowncenterapts.com/) TownCenter in the heart of **Raleigh!** + [Show map of Raleigh](#)"
  - Ad 3: "**Apartment Finder Raleigh** - Search **Raleigh Apt Deals**. [www.apartmentfinder.com/Raleigh](http://www.apartmentfinder.com/Raleigh) View Photos, Virtual Tours, & More!"
  - Ad 4: "3 **Bedroom Raleigh Apartments** for Rent - Find 3 Bed"
- Right column of ads:
  - Ad 1: "Looking For a 3 Bdrm Apt? [www.forrent.com/3Bedroom](http://www.forrent.com/3Bedroom) Well, This is Your Lucky Day... We Have The Most 3 Bdrms On"
  - Ad 2: "3 **Bedroom Apartment For I** [www.rentbits.com/](http://www.rentbits.com/) Search **Apartment** Rentals Fre Fast. Free. No Login Required."
  - Ad 3: "**Apts With Paid Utilities** [www.apartmentlist.com/Raleig](http://www.apartmentlist.com/Raleig) Cheap Studio, 1,2 & 3 BR **Apartments.** Search by Price, City, Zip & Moi"

# Review of a Few Terms



Apartment List http://find.apartmentlist.com/srp?city=raleigh&state=nc&gclid=...





(/)

Like 1147 likes. Sign Up to see what your friends like. 17 Find your Apartment Deal Today!

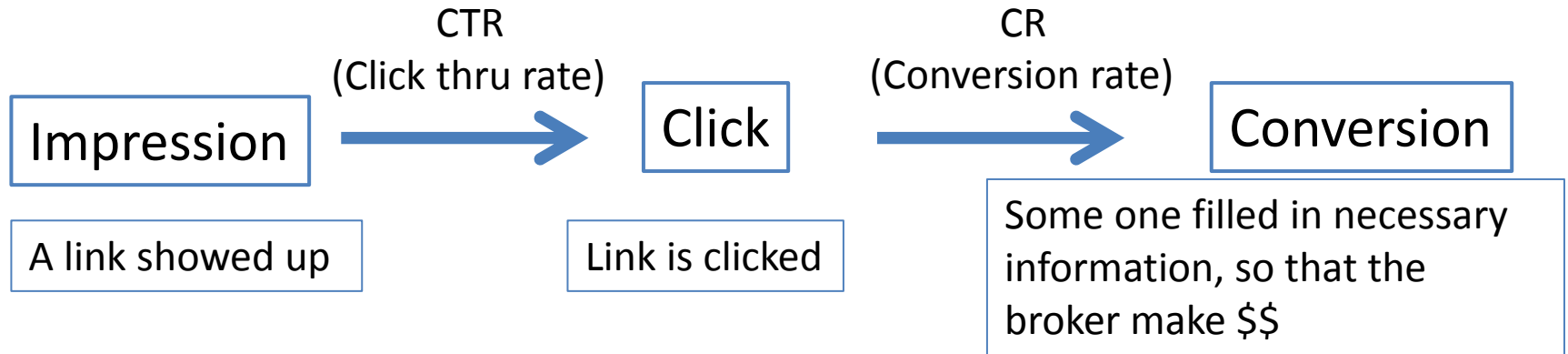
LOCATION: Raleigh, NC      "New York, NY", "43438"      MIN PRICE: 200      MAX PRICE: 10000      BEDROOMS: Studio+      SORT BY: [ ]

237 Apartment Communities near Raleigh, NC

Results within Raleigh, NC

|   |   |           |       |            |       |            |         |
|---|---|-----------|-------|------------|-------|------------|---------|
|   | <b>Bell Falls River</b> (/ldp?city=raleigh&id=941&zip=27614) 1302 Rio Valley Dr, Raleigh, NC 27614  |           |       |            |       |            |         |
|   | <table border="0"> <tr><td>1 Bedroom</td><td>\$865</td></tr> <tr><td>2 Bedrooms</td><td>\$954</td></tr> <tr><td>3 Bedrooms</td><td>\$1,104</td></tr> </table> <p style="text-align: right;">Photos &amp; Details (/ldp?city=raleigh&amp;id=941&amp;zip=27614)</p>   | 1 Bedroom | \$865 | 2 Bedrooms | \$954 | 3 Bedrooms | \$1,104 |
| 1 Bedroom   | \$865   |           |       |            |       |            |         |
| 2 Bedrooms  | \$954   |           |       |            |       |            |         |
| 3 Bedrooms  | \$1,104   |           |       |            |       |            |         |
|  | <b>Remington on the Green</b> (/ldp?city=raleigh&id=931&zip=27604) 2904 Calvary Drive, Raleigh, NC 27604  |           |       |            |       |            |         |
|   | <table border="0"> <tr><td>1 Bedroom</td><td>\$664</td></tr> <tr><td>2 Bedrooms</td><td>\$772</td></tr> </table> <p style="text-align: right;">Photos &amp; Details (/ldp?city=raleigh&amp;id=931&amp;zip=27604)</p>  | 1 Bedroom | \$664 | 2 Bedrooms | \$772 |            |         |
| 1 Bedroom   | \$664   |           |       |            |       |            |         |
| 2 Bedrooms  | \$772   |           |       |            |       |            |         |
|  | <b>Providence at Brier Creek</b> (/ldp?city=raleigh&id=64672&zip=27617) 10100 Donerall Way, Raleigh, NC 27617   |           |       |            |       |            |         |
|   | <table border="0"> <tr><td>1 Bedroom</td><td>\$725</td></tr> <tr><td>2 Bedrooms</td><td>\$805</td></tr> <tr><td>3 Bedrooms</td><td>\$1,090</td></tr> </table> <p style="text-align: right;">Photos &amp; Details (/ldp?city=raleigh&amp;id=64672&amp;zip=27617)</p> | 1 Bedroom | \$725 | 2 Bedrooms | \$805 | 3 Bedrooms | \$1,090 |
| 1 Bedroom   | \$725   |           |       |            |       |            |         |
| 2 Bedrooms  | \$805   |           |       |            |       |            |         |
| 3 Bedrooms  | \$1,090   |           |       |            |       |            |         |
|  | <b>Furnished Studio - Raleigh - North Raleigh</b> (/ldp?city=raleigh&id=207309&zip=27609) 811 Wake Towne Dr., Raleigh, NC 27609   |           |       |            |       |            |         |
|   | <table border="0"> <tr><td>Studio</td><td>\$945</td></tr> </table> <p style="text-align: right;">Photos &amp; Details (/ldp?city=raleigh&amp;id=207309&amp;zip=27609)</p>   | Studio    | \$945 |            |       |            |         |
| Studio  | \$945   |           |       |            |       |            |         |

# Review of a Few Terms



Bell Falls River  
1302 Rio Valley Dr, Raleigh, NC 27614

Contact Property (919) 426-2911

Full Name (required)  
xyz

Email Address  
xhys

Phone Number  
4004

Specify a unit type or ask a general question.  
too many

Send me email alerts for apartments in my area

# Simple Economics for Broker

- Profit = revenue – cost
  - = leads x \$ per lead – clicks x CPC
  - = clicks x CR x \$ per lead – clicks x CPC
  - = clicks x (CR x \$per lead – CPC)
  - = clicks x (RPC – CPC)
    - RPC = revenue per click (= CR x \$per lead)
- **Objective:** maximize Profit
- **Control variable:** maxCPC (maximum amount willing to pay for CPC)
- Assume no budget cap (*simplification*)

# Other Queries Not Covered in This Talk

- Which keyword?
- When to bid?
- How to write the ad?
- How to take advantage of user's profile?
- How to group keyword?

# Maximize Profit

- Large maxCPC (higher bid)
  - Higher position (always)
  - More clicks (assumed)
  - Higher CPC (always)
- An example
  - Same keyword
  - Same web site
  - RPC = 9 (assumed fixed)
- Optimal maxCPC  $\in [5,7)$
- Relates to ICC (incremental CPC)

| Position | Current bids | Clicks | Profit        |
|----------|--------------|--------|---------------|
| 1        | \$10         | 100    | $(9-9)100=0$  |
| 2        | \$9          | 90     | $(9-7)90=180$ |
| 3        | \$7          | 70     | $(9-5)70=280$ |
| 4        | \$5          | 50     | $(9-3)50=300$ |
| 5        | \$3          | 30     | $(9-2)30=210$ |



# Outline

- I. Problem Description
- II. Game theoretical aspect of the bidding problem that we will consider
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## II. Gaming Aspect in AdWords

- Advertisers (bidders):  $1, \dots, N$
- Slots in AdWords:  $1, \dots, K, K < N$
- $i$ -th bidder, with RPC (aka expected return)  $v^i$
- Descending current bids:  $b_1 > b_2 > \dots > b_K$ 
  - $b_i$  = bid amount by bidder at position (slot)  $i$
- Clicks for the  $i$ th bidder: WLOG,  
$$c_1^i \geq c_2^i \geq \dots \geq c_K^i$$

See justification next page.

# Continue with Games

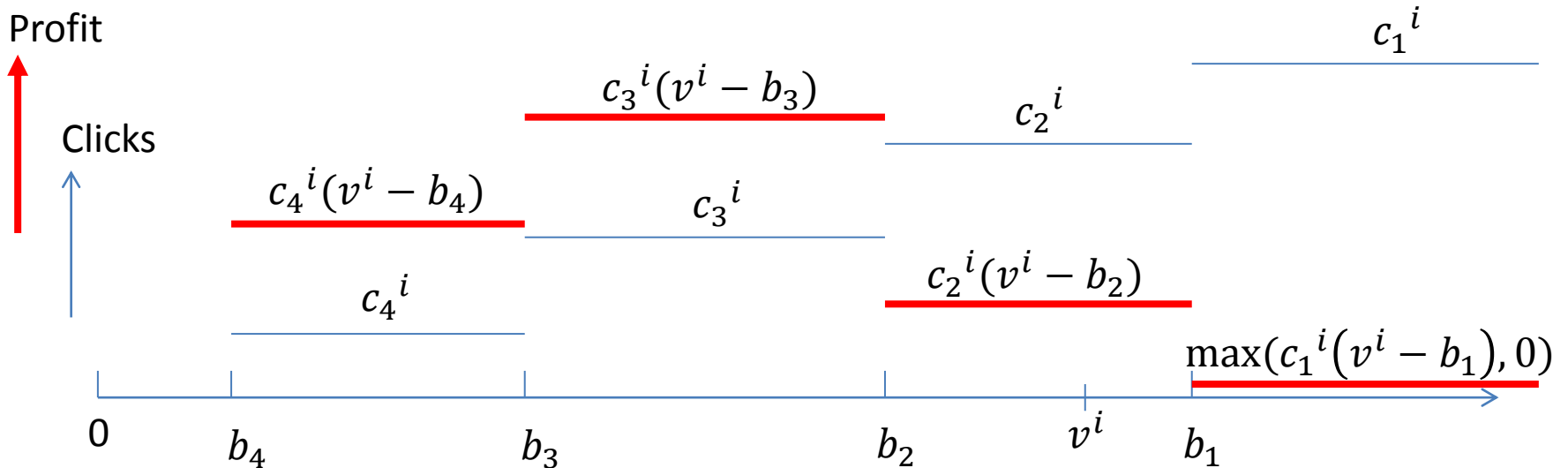
- Utility of bidder  $i$ : if bidder  $i$  bids  $b_k$  (i.e., he/she is the  $k$ th highest bid), then his/her utility (i.e., profit) is

$$(v^i - b_{k+1})c_k^i$$

- Bidders want to maximize utilities
- Justification of non-increasing  $c_k^i$ 
  - If  $\exists \alpha, c_\alpha^i < c_{\alpha+1}^i$ ,
  - then  $(v^i - b_{\alpha+1})c_\alpha^i < (v^i - b_{\alpha+2})c_{\alpha+1}^i$ .
  - Having  $c_\alpha^i = c_{\alpha+1}^i$  preserves the above inequality, won't change the outcome of the maximization problem.

# Illustration: utility versus maxCPC

Piecewise constant utility/click function



Bid amount (i.e., maxCPC) by the  $i$ th advertiser

# Recap of the Game

- Every bidder knows
  - current bids:  $b_1 > b_2 > \dots > b_K$
  - clicks for all:  $c_1^i \geq c_2^i \geq \dots \geq c_K^i$
- Every bidder choose maxCPC (given  $v^i$ ), so that she can achieves  $k^*$  that maximizes utility  $(v^i - b_{k+1})c_k^i$  as a function of  $k$
- Q: does *equilibrium* achievable?

# Nash Equilibriums

- Notations
  - $\mathbf{b} = (b^1, b^2, \dots, b^N)$ ; note the difference betwn  $b^i$  and  $b_i$
  - $\mathbf{b}^{-i} = (b^1, \dots, b^{i-1}, b^{i+1}, \dots, b^N)$ ; exclude  $b^i$
- **Best response of bidder  $i$ :**
  - $M^i(\mathbf{b}^{-i}) =$  given  $\mathbf{b}^{-i}$ , the set of values of  $b^i$  such that the utility of bidder  $i$  is maximized
- **Nash equilibrium:** a strategy profile  $\mathbf{b}$  such that  $\forall i, b^i \in M^i(\mathbf{b}^{-i})$ .

# Property of Equilibriums

- Does it exist?
  - Yes. Can give examples...
- Is it reasonable?
  - Need to define reasonableness

# Vulnerability of Nash Equilibriums

- $O^i(\mathbf{b})$  bidder  $i$ 's position in the descending bid queue
- **Output truthful (OT)** property of a position auction:
  - $\forall$  equilibrium set  $\mathbf{e}$ , and  
 $\forall i, O^i(\mathbf{e}) = O^i(v^1, \dots, v^N)$ , the auction is output truthful
- Counterexample in Bu, Deng, and Qi (2008)



# Designing Objective of a Bidding Strategy

- A strategy that is available to all bidders
- Those who follow maximize their utilities
- Those who don't will not negatively affect others
- Violators don't maximize their utilities
- *Equilibrium exists and unique*

# “Forward Looking” Strategy

- The higher bidder  $i$  bids, the higher a slot she can get in the next step
- Bid as high as possible in the set  $M^i(\mathbf{b}^{-i})$  -- best response
- Control the risks of decreasing their own payoffs by the affected bidders' next optimal moves (kind of technical)

# A Derived “Forward Looking” Strategy

- [Bu *et al*, 2008] for bidder  $i$ , given  $\mathbf{b}^{-i}$ , suppose  $k$  is the optimal position that maximizes her utility, this bidder’s next bid is

$$F^i(\mathbf{b}^{-i}) = \begin{cases} v^i - \frac{c_k}{c_{k-1}}(v^i - b_{k+1}), & 2 \leq k \leq K, \\ v^i, & k = 1 \text{ or } k > K. \end{cases}$$

- Here  $c_k$  and  $c_{k-1}$  are the clicks of the bidder who occupies slot  $k - 1$

# Forward Looking Equilibrium

- A forward looking response function based *equilibrium* is a strategy profile  $\mathbf{b}$  such that  $\forall i, 1 \leq i \leq N, b^i = F^i(\mathbf{b}^{-i})$
- That is, every bidder follows the forward looking scheme
- The equilibriums exist
- The position auction is **output truthful** under the **forward looking best response** scheme; i.e., the corresponding equilibrium is always output truthful

# Proof of the “output truthfulness”

- Using contradiction, if OT does not hold, (in an equilibrium,) there must exist a pair of adjacent slots  $k, k + 1$  and the bidder  $i$  on slot  $k$  and the bidder  $j$  on slot  $k + 1$  such that  $v^j > v^i$
- Let  $u_k^i$  denote the utility of bidder  $i$  at slot  $k$ , and  $u_{k+1}^i$  the utility at slot  $k + 1$ , the inequalities on the next page establishes a contradiction: bidder would prefer slot  $k + 1$

# Inequality of the OTness

- $u_k^i = (v^i - b_{k+1})c_k$   
 $= \left( v^i - \left( v^j - \frac{c_{k+1}}{c_k} (v^j - b_{k+2}) \right) \right) c_k$   
 $= (v^i - b_{k+2})c_{k+1} + (c_k - c_{k+1})(v^i - v^j)$   
 $< (v^i - b_{k+2})c_{k+1}$   
 $= u_{k+1}^i$
- The above contradicts to the equilibrium

# Uniqueness of FL Equilibrium

- The position auction has a unique forward looking Nash equilibrium
- Sketch of the proof:
  - Output Truthfulness
  - The Forward Looking best response formula

# Convergence of Forward Looking Strategy

- Does it converge? Yes, *hopefully*...
- If at every time, one bidder bids, and this bidder is randomly chosen, then the forward looking strategy will eventually converge to its equilibrium.



# More on Forward Looking Equilibrium

- FL schemes maximizes bidders who follow it;
- Follower won't be punished by actions from those non-followers;
- non-followers will not maximize their utilities.

# Outline

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# III. Implementation Issues

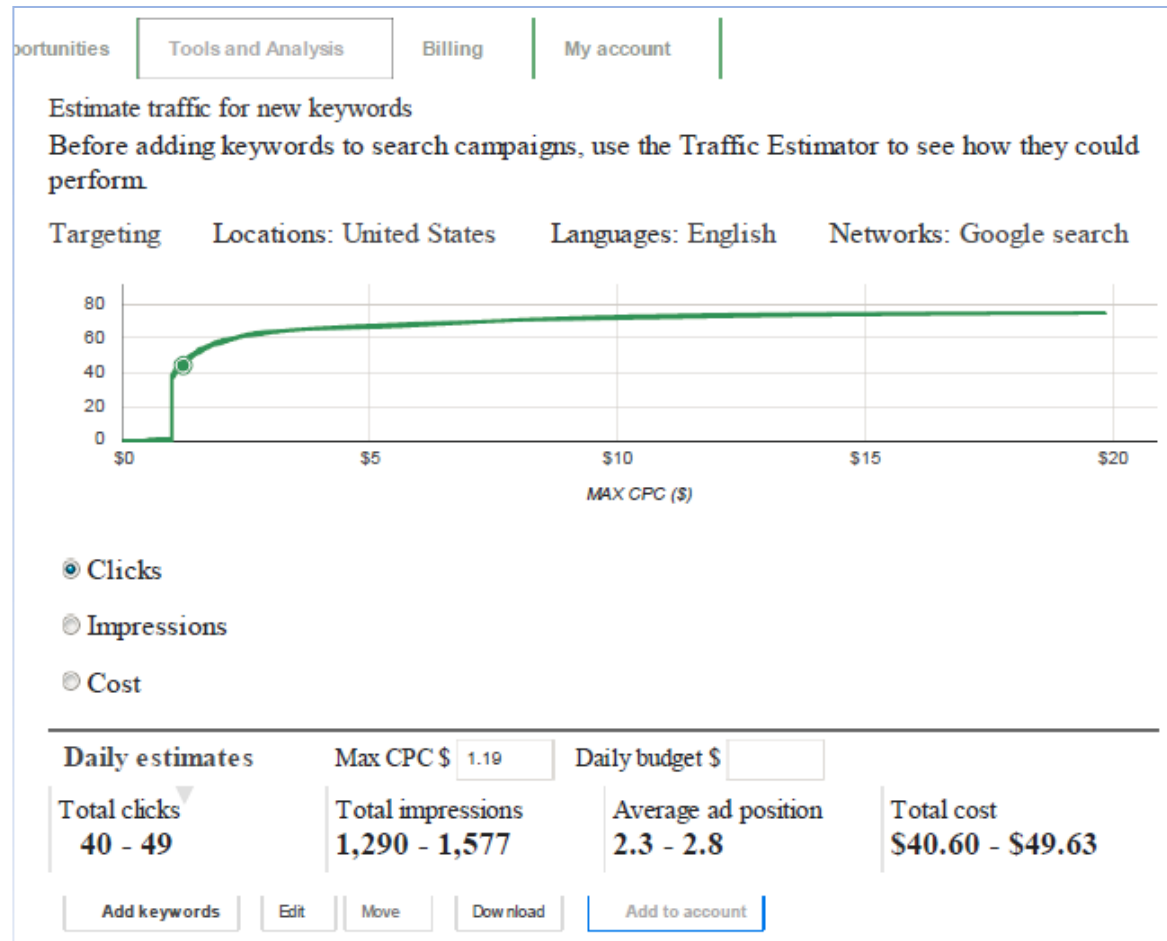
- Recall the forward looking strategy
  - Find the best response slot  $k$ , then do the following

$$F^i(\mathbf{b}^{-i}) = \begin{cases} v^i - \frac{c_k}{c_{k-1}} (v^i - b_{k+1}), & 2 \leq k \leq K, \\ v^i, & k = 1 \text{ or } k > K. \end{cases}$$

- Need to know:
  - $c_k$  and  $c_{k-1}$  are the clicks of the bidder  $j$  who occupies slot  $k - 1$
  - $b_{k+1}$ , the immediate lower bid; This is the current CPC!
- Possible solution: use AdWords traffic estimator

# AdWords: Traffic Estimator

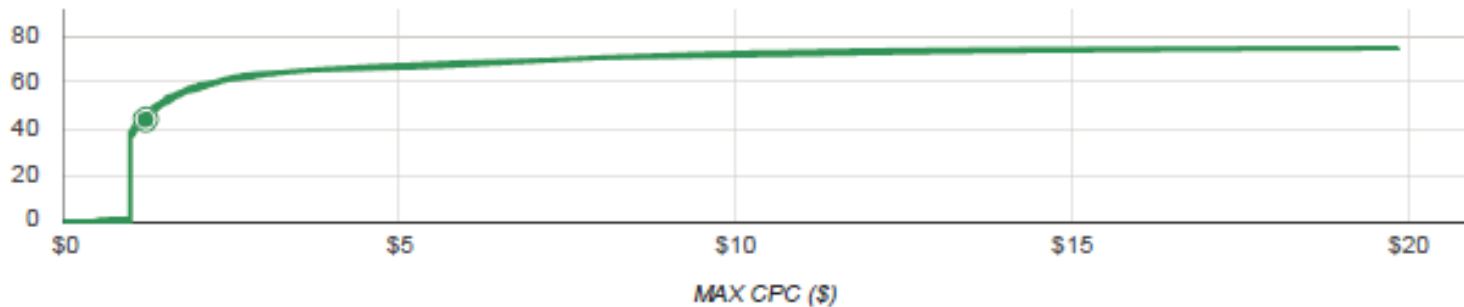
- Given a key word, Traffic Estimator tells how it could perform



### Estimate traffic for new keywords

Before adding keywords to search campaigns, use the Traffic Estimator to see how they could perform

Targeting    Locations: United States    Languages: English    Networks: Google search



- Clicks
- Impressions
- Cost

#### Daily estimates

Max CPC \$

Daily budget \$

Total clicks  
**40 - 49**

Total impressions  
**1,290 - 1,577**

Average ad position  
**2.3 - 2.8**

Total cost  
**\$40.60 - \$49.63**

Add keywords

Edit

Move

Download

Add to account

# Use Adwords Traffic Estimator for Inference

- Estimating  $c_k$  and  $c_{k-1}$ :
  - ATE gives average positions and a range of clicks
  - Need statistical model to estimate  $c_k$ 's
- Getting  $b_{k+1}$ :
  - Current CPC paid by this bidder
  - Time varying
- Big question: can we really trust ATE to perform the above tasks? (do not know...)
- If there is a statistically consistent way to estimate  $c_k$ , forward looking converge to equilibrium with high probability

# Other Considerations

- We assumed known RPC; however in reality, the conversion rate (CR) is highly stochastic
- One may infer  $c_k$  through knowledge of the bidders (their web sites are observable) and its own experience; for example, similar web sites likely have similar  $c_k$ 's
- In fact, many online testimonies say that you can learn by trying AdWords – see @ youtube

# IV. Conclusion

- We study the keyword bidding problem at AdWords
- A strategy named “forward looking best response” is a promising way to bid:
  - It can be made available publicly
  - Maximize utility
  - Immune to adversary bids
- There remain some statistical estimation problems unsolved
- Haven’t been tested in reality
- Stochasticity can be another issue