Considerations of Modeling in Keyword Bidding (Google:AdWords)

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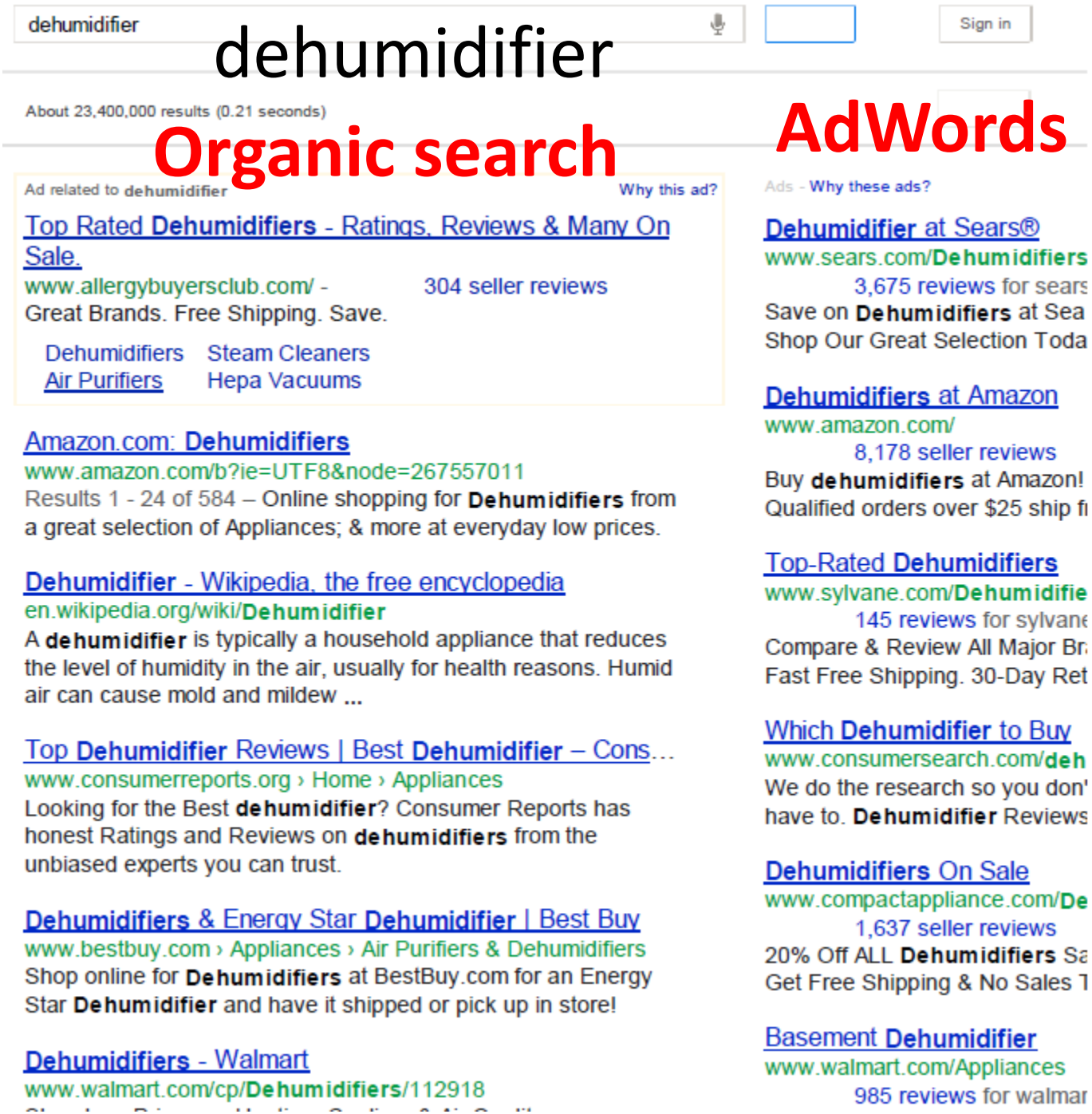
August 8, 2012

Outline

I. Problem Description
II. Game theoretical aspect of the bidding problem that we are considering
III. Statistical Implementation
IV. Conclusion

I. Introduction

- We consider the modeling issue in AdWords
- An equilibrium strategy is derived, which we argue will be the foundation of statistical modeling
- What is AdWords: A tool at Google that allow users to bid for advertising positions at google.com



dehumidifier

Organic search

AdWords

Hotel raleigh NC

hotel raleigh nc

Sign in

About 6,230,000 results (0.35 seconds)

Ads related to hotel raleigh nc — Why these ads?

Washington Duke Inn - Located at Duke University
www.washingtondukeinn.com/
3 Night Special from $119. Book Now

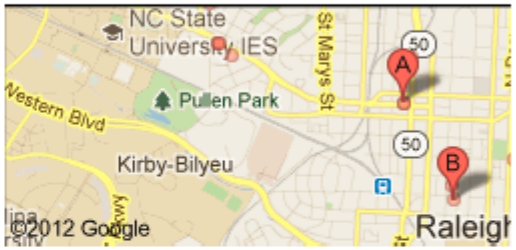
Clarion Hotel® Raleigh NC - Official site | ClarionHotel.com
www.clarionhotel.com/
Hotel near Conventio Center. Best Internet Rates.
Summer Promo - Earn A $50 Gift Card - Want Hotel Deals? Find Them Here

Sponsored

Book hotels in Raleigh
www.google.com/hotelfinder
☑ 2-star hotels from $42 ☑ 4-star hotels from $179
☑ 3-star hotels from $61 ☐ 5-star hotels
Book hotels

Downtown Raleigh Hotels, Ralei… 320 Hillsborough Street Raleigh

Map for **hotel raleigh nc**



Ads - Why these ads?

HYATT house® Raleigh
house.hyatt.com/
All Suites **Hotels** with a Full Kitchen. Free Breakfast & Intern
122,198 people +1'd or follow Hyatt

Hotels in Raleigh, NC
www.expedia.com/Raleigh_No
436 reviews for expedia
Summer Sale: **Hotels** up to 30%
Save on **Raleigh, NC Hotels**.
295,202 people +1'd or follow Expedia

A Simplified Version of AdWords Bidding

- Sorted (descending) bids $b_j, j = 1,2,\cdots,N$, of N potential advertisers: $b_1 > b_2 > \cdots > b_N$
- Positions and Cost per Click by AdWords
- GSP: generalized second price

Position	Bid price	Actual price (i.e., CPC)
1	b_1	b_2+ 1 cent
2	b_2	b_3+ 1 cent
3	b_3	b_4+ 1 cent
4	b_4	b_5+ 1 cent
5	b_5	Assume out of space

Actual Scheme: Incorporate a Quality Score

- The quality score (aka, AdRank) that depends on relevance, past click through rate, landing page, etc.
- Purpose: integrating web page quality, user experience, user satisfaction
- Prevent: bad, irrelevant ads goes to top positions by paying more (customer satisfaction)

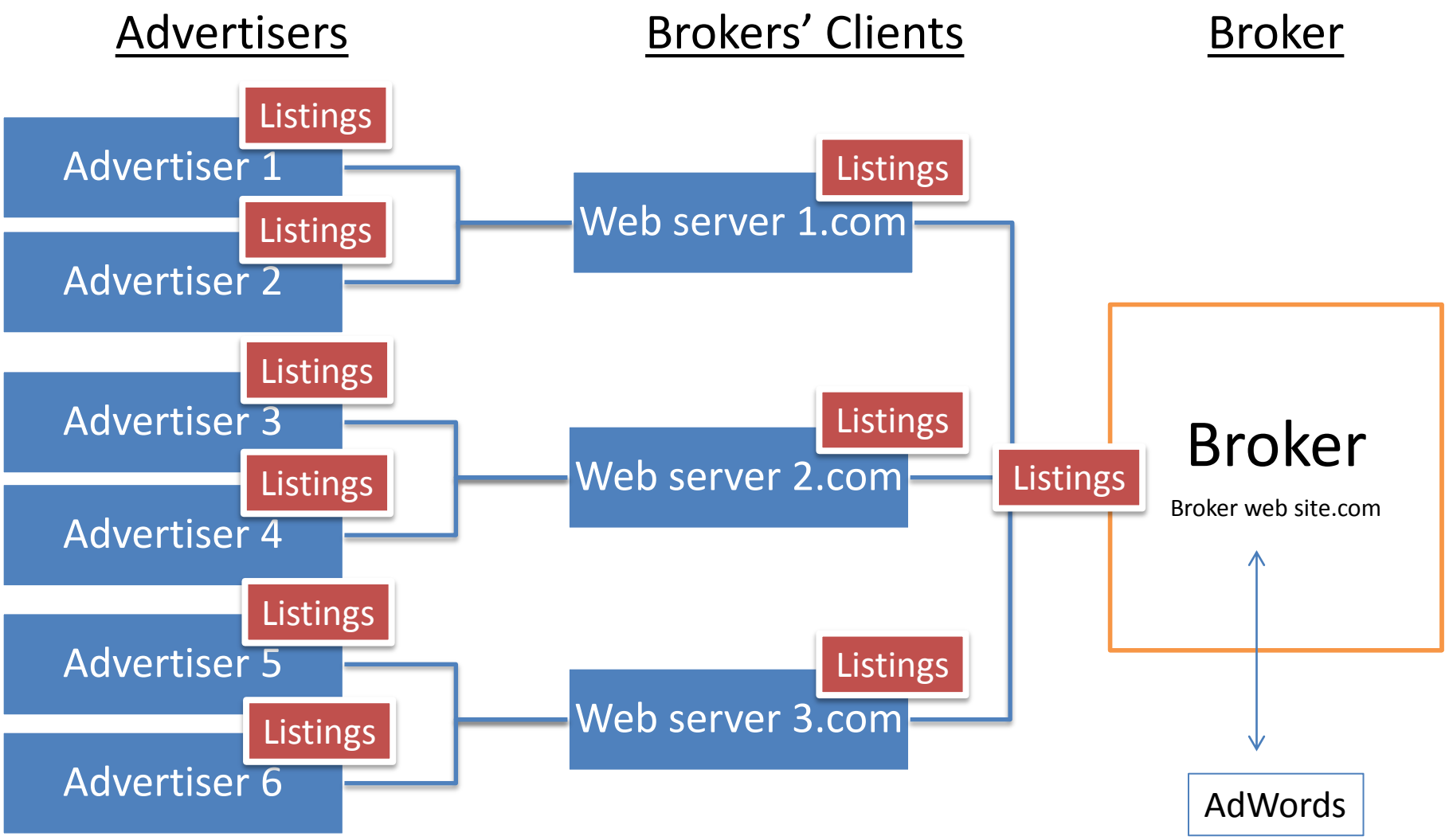
An Example

- Bidding with quality scores (rank according to $b_j q_j$)

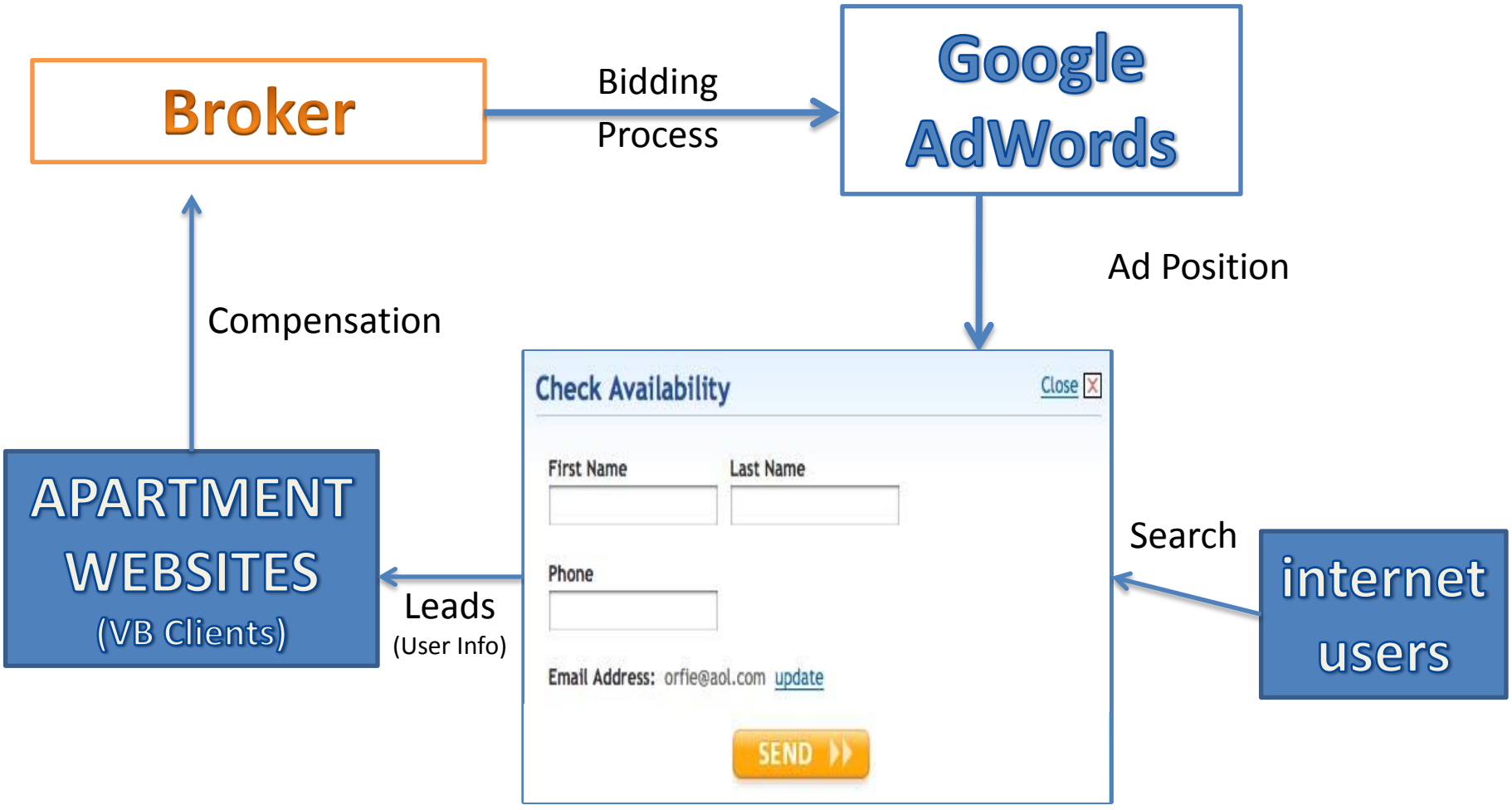
Position	Bid price	Quaity score	Multiply	Actual price (i.e., CPC)
1	b_1	q_1	$b_1 q_1$	$b_2 q_2 / q_1$ + 1 cent
2	b_2	q_2	$b_2 q_2$	$b_3 q_3 / q_2$ + 1 cent
3	b_3	q_3	$b_3 q_3$	$b_4 q_4 / q_3$ + 1 cent
4	b_4	q_4	$b_4 q_4$	$b_5 q_5 / q_4$ + 1 cent
5	b_5	q_5	$b_5 q_5$	Assume out of space

- Assume: $b_1 q_1 > b_2 q_2 > \cdots > b_N q_N$
- Note: the order of advertisers may change from the previous example

Problem Description



Broker & Google AdWords



Review of a Few Terms

Impression

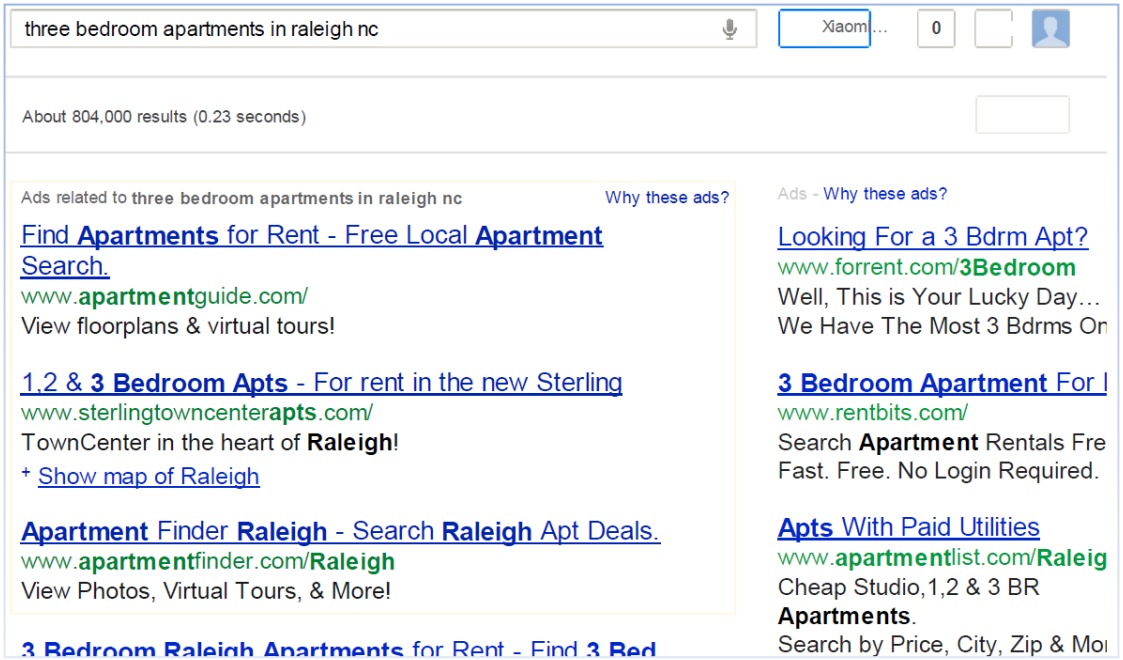
A link showed up

Click

Link is clicked

Conversion

Some one filled in necessary information, so that the broker make $$



Review of a Few Terms

CTR
(Click thru rate)

Impression → Click

Conversion

A link showed up

Link is clicked

Some one filled in necessary
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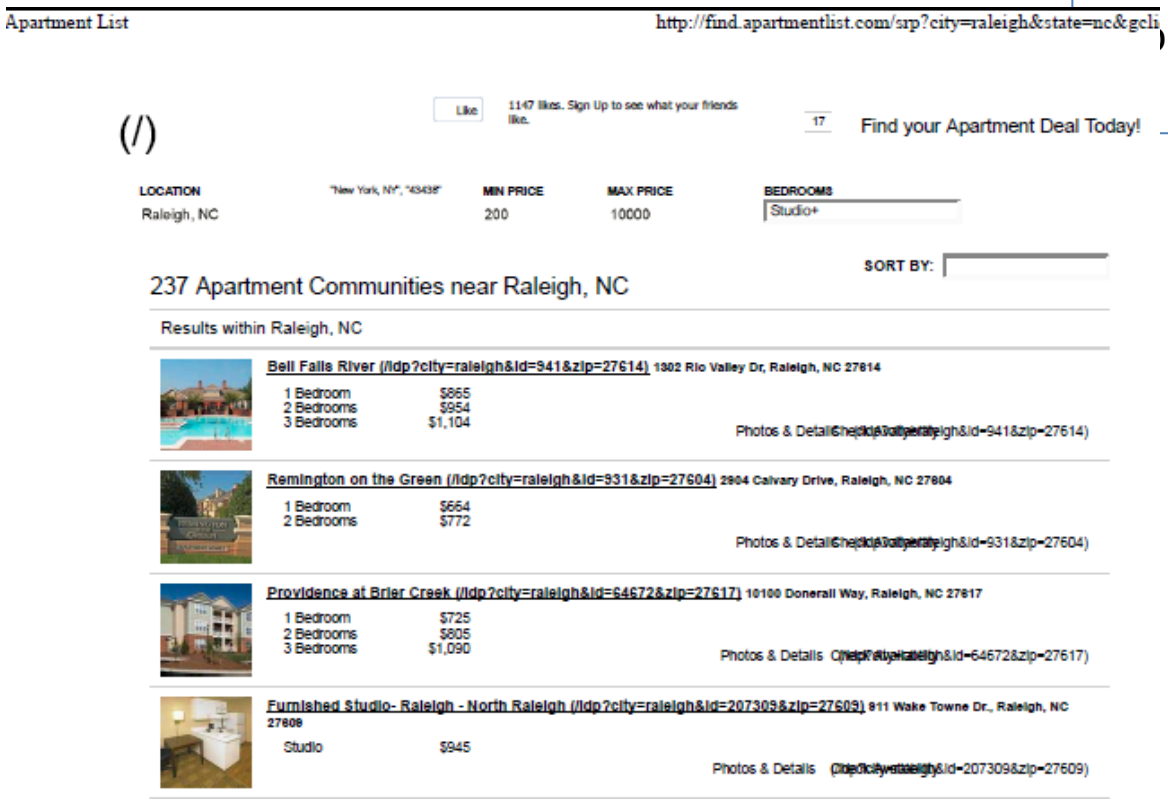

Review of a Few Terms

CTR
(Click thru rate)

CR
(Conversion rate)

Impression → **Click** → **Conversion**

A link showed up

Link is clicked

Some one filled in necessary information, so that the broker make $$

Bell Falls River
1302 Rio Valley Dr, Raleigh, NC 27614

Contact Property (919) 426-2911

Full Name (required)
xyz

Email Address
xhys

Phone Number
4004

Specify a unit type or ask a general question.
too many

Send me email alerts for apartments in my area

Simple Economics for Broker

- Profit = revenue – cost
  = leads x $ per lead – clicks x CPC
  = clicks x CR x $ per lead – clicks x CPC
  = clicks x (CR x $per lead – CPC)
  = clicks x (RPC – CPC)
  - RPC = revenue per click (= CR x $per lead)
- **Objective**: maximize Profit
- **Control variable**: maxCPC (maximum amount willing to pay for CPC)
- Assume no budget cap (*simplification*)

Other Queries Not Covered in This Talk

- Which keyword?
- When to bid?
- How to write the ad?
- How to take advantage of user's profile?
- How to group keyword?

Maximize Profit

- Large maxCPC (higher bid)
  - Higher position (always)
  - More clicks (assumed)
  - Higher CPC (always)
- An example
  - Same keyword
  - Same web site
  - RPC = 9 (assumed fixed)
- Optimal maxCPC $\in [5,7)$
- Relates to ICC (incremental CPC)

Position	Current bids	Clicks	Profit
1	$10	100	(9-9)100=0
2	$9	90	(9-7)90=180
3	$7	70	(9-5)70=280
4	$5	50	(9-3)50= **300**
5	$3	30	(9-2)30=210

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II. Gaming Aspect in AdWords

- Advertisers (bidders): $1, \cdots, N$
- Slots in AdWords: $1, \cdots, K, K < N$
- i-th bidder, with RPC (aka expected return) v^i
- Descending current bids: $b_1 > b_2 > \cdots > b_K$
  - b_i = bid amount by bidder at position (slot) i
- Clicks for the ith bidder: WLOG,

$$c_1{}^i \geq c_2{}^i \geq \cdots \geq c_K{}^i$$

See justification next page.

Continue with Games

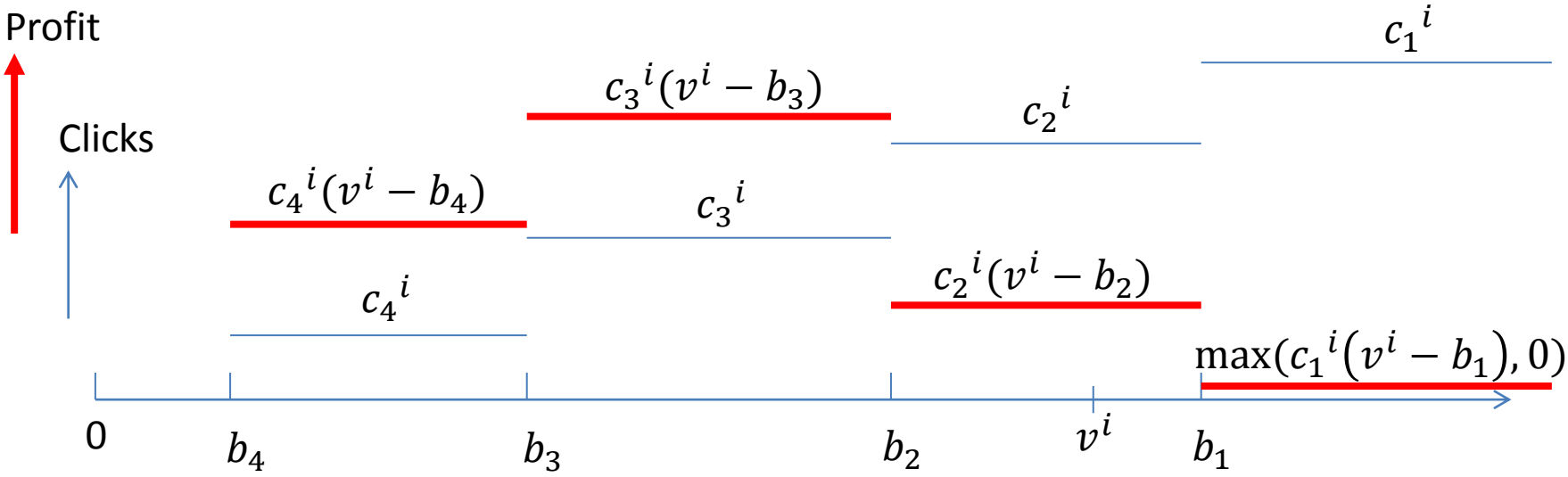
- Utility of bidder i: if bidder i bids b_k(i.e., he/she is the kth highest bid), then his/her utility (i.e., profit) is

$$\left(v^i - b_{k+1}\right){c_k}^i$$

- Bidders want to maximize utilities
- Justification of non-increasing ${c_k}^i$
  - If $\exists \alpha, {c_\alpha}^i < {c_{\alpha+1}}^i$,
  - then $\left(v^i - b_{\alpha+1}\right){c_\alpha}^i < \left(v^i - b_{\alpha+2}\right){c_{\alpha+1}}^i$.
  - Having ${c_\alpha}^i = {c_{\alpha+1}}^i$ preserves the above inequality, won't change the outcome of the maximization problem.

Illustration: utility versus maxCPC

Piecewise constant utility/click function



Bid amount (i.e., maxCPC) by the ith advertiser

Recap of the Game

- Every bidder knows
  - current bids: $b_1 > b_2 > \cdots > b_K$
  - clicks for all: ${c_1}^i \geq {c_2}^i \geq \cdots \geq {c_K}^i$
- Every bidder choose maxCPC (given v^i), so that she can achieves k^* that maximizes utility $\left(v^i - b_{k+1}\right){c_k}^i$ as a function of k
- Q: does *equilibrium* achievable?

Nash Equilibriums

- Notations
  - $\boldsymbol{b} = (b^1, b^2, \ldots, b^N)$; note the difference betwn b^i and b_i
  - $\boldsymbol{b}^{-i} = \left(b^1, \ldots, b^{i-1}, b^{i+1}, \ldots, b^N\right)$; exclude b^i
- **Best response** of bidder i:
  - $M^i\left(\boldsymbol{b}^{-i}\right) =$ given $\boldsymbol{b}^{-i}$, the set of values of b^i such that the utility of bidder i is maximized
- **Nash equilibrium**: a strategy profile $\boldsymbol{b}$ such that $\forall i, b^i \in M^i\left(\boldsymbol{b}^{-i}\right)$.

Property of Equilibriums

- Does it exist?
  - Yes. Can give examples…
- Is it reasonable?
  - Need to define reasonableness

Vulnerability of Nash Equilibriums

- $O^i(\boldsymbol{b})$ bidder i's position in the descending bid queue
- **Output truthful (OT)** property of a position auction:
  - $\forall$ equilibrium set $\boldsymbol{e}$, and $\forall i, O^i(\boldsymbol{e}) = O^i(v^1, \ldots, v^N)$, the auction is output truthful
- Counterexample in Bu, Deng, and Qi (2008)

Designing Objective of a Bidding Strategy

- A strategy that is available to all bidders
- Those who follow maximize their utilities
- Those who don't will not negatively affect others
- Violators don't maximize their utilities
- *Equilibrium exists and unique*

“Forward Looking” Strategy

- The higher bidder i bids, the higher a slot she can get in the next step
- Bid as high as possible in the set $M^i(\boldsymbol{b}^{-i})$ -- best response
- Control the risks of decreasing their own payoffs by the affected bidders’ next optimal moves (kind of technical)

A Derived "Forward Looking" Strategy

- [Bu *et al*, 2008] for bidder i, given $\boldsymbol{b}^{-i}$, suppose k is the optimal position that maximizes her utility, this bidder's next bid is

$$F^i\left(\boldsymbol{b}^{-i}\right) = \begin{cases} v^i - \dfrac{c_k}{c_{k-1}}\left(v^i - b_{k+1}\right), & 2 \le k \le K, \\ v^i, & k = 1 \text{ or } k > K. \end{cases}$$

- Here c_k and c_{k-1} are the clicks of the bidder who occupies slot $k-1$

Forward Looking Equilibrium

- A forward looking response function based *equilibrium* is a strategy profile $\boldsymbol{b}$ such that $\forall i, 1 \leq i \leq N, b^i = F^i\left(\boldsymbol{b}^{-i}\right)$
- That is, every bidder follows the forward looking scheme
- The equilibriums exist
- The position auction is **output truthful** under the **forward looking best response** scheme; i.e., the corresponding equilibrium is always output truthful

Proof of the "output truthfulness"

- Using contradiction, if OT does not hold, (in an equilibrium,) there must exist a pair of adjacent slots $k, k+1$ and the bidder i on slot k and the bidder j on slot $k+1$ such that $v^j > v^i$
- Let ${u_k}^i$ denote the utility of bidder i at slot k, and ${u_{k+1}}^i$ the utility at slot $k+1$, the inequalities on the next page establishes a contradiction: bidder would prefer slot $k+1$

Inequality of the OTness

- $u_k{}^i = (v^i - b_{k+1})c_k$

$$= \left(v^i - \left(v^j - \frac{c_{k+1}}{c_k}\left(v^j - b_{k+2}\right)\right)\right)c_k$$

$$= \left(v^i - b_{k+2}\right)c_{k+1} + (c_k - c_{k+1})(v^i - v^j)$$

$$< (v^i - b_{k+2})c_{k+1}$$

$$= u_{k+1}{}^i$$

- The above contradicts to the equilibrium

Uniqueness of FL Equilibrium

- The position auction has a unique forward looking Nash equilibrium
- Sketch of the proof:
  - Output Truthfulness
  - The Forward Looking best response formula

Convergence of Forward Looking Strategy

- Does it converge? Yes, *hopefully*…
- If at every time, one bidder bids, and this bidder is randomly chosen, then the forward looking strategy will eventually converge to its equilibrium.

More on Forward Looking Equilibrium

- FL schemes maximizes bidders who follow it;
- Follower won't be punished by actions from those non-followers;
- non-followers will not maximize their utilities.

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III. Implementation Issues

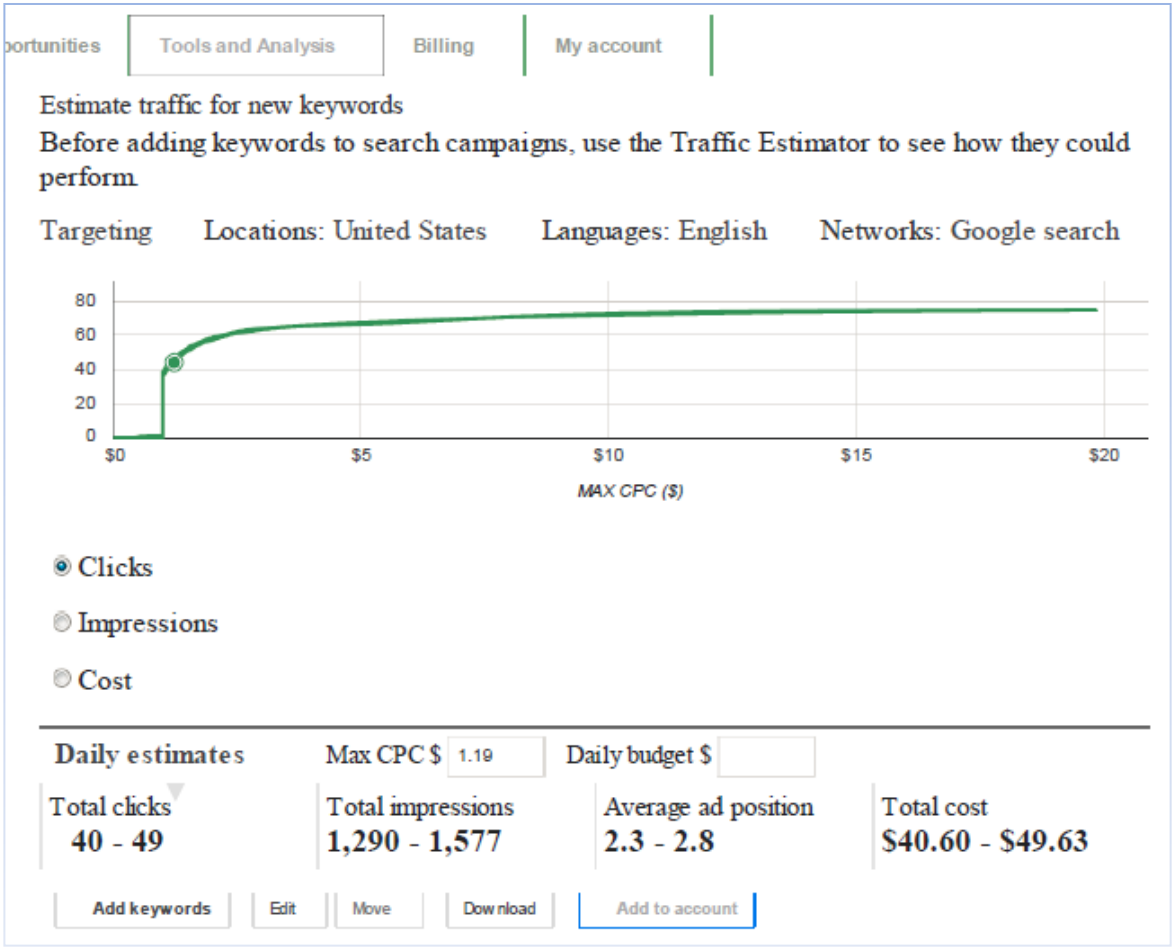
- Recall the forward looking strategy
  - Find the best response slot k, then do the following

$$F^i(\boldsymbol{b}^{-i}) = \begin{cases} v^i - \dfrac{c_k}{c_{k-1}}\left(v^i - b_{k+1}\right), & 2 \le k \le K, \\ v^i, & k = 1 \text{ or } k > K. \end{cases}$$

- Need to know:
  - c_k and c_{k-1} are the clicks of the bidder j who occupies slot $k-1$
  - b_{k+1}, the immediate lower bid; This is the current CPC!
- Possible solution: use AdWords traffic estimator

AdWords: Traffic Estimator

- Given a key word, Traffic Estimator tells how it could perform

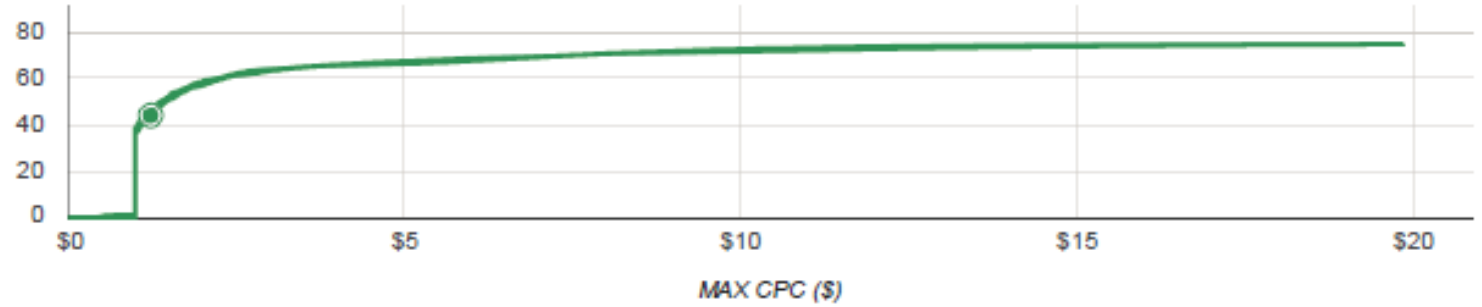


portunities | Tools and Analysis | Billing | My account

Estimate traffic for new keywords

Before adding keywords to search campaigns, use the Traffic Estimator to see how they could perform

Targeting Locations: United States Languages: English Networks: Google search



- (•) Clicks
- ( ) Impressions
- ( ) Cost

Daily estimates Max CPC $ 1.19 Daily budget $

Total clicks	Total impressions	Average ad position	Total cost
40 - 49	1,290 - 1,577	2.3 - 2.8	$40.60 - $49.63

Add keywords | Edit | Move | Download | Add to account

Use Adwords Traffic Estimator for Inference

- Estimating c_k and c_{k-1}:
  - ATE gives average positions and a range of clicks
  - Need statistical model to estimate c_k's
- Getting b_{k+1}:
  - Current CPC paid by this bidder
  - Time varying
- Big question: can we really trust ATE to perform the above tasks? (do not know…)
- If there is a statistically consistent way to estimate c_k, forward looking converge to equilibrium with high probability

Other Considerations

- We assumed known RPC; however in reality, the conversion rate (CR) is highly stochastic
- One may infer c_k through knowledge of the bidders (their web sites are observable) and its own experience; for example, similar web sites likely have similar c_k's
- In fact, many online testimonies say that you can learn by trying AdWords – see @ youtube

IV. Conclusion

- We study the keyword bidding problem at AdWords
- A strategy named “forward looking best response” is a promising way to bid:
  - It can be made available publicly
  - Maximize utility
  - Immune to adversary bids
- There remain some statistical estimation problems unsolved
- Haven’t been tested in reality
- Stochasticity can be another issue